

Geometric Scaling at RHIC and LHC

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We present a new phenomenological model of the dipole scattering amplitude to demonstrate that the RHIC data for hadron production in d -Au collisions for all available rapidities are compatible with geometric scaling, just like the small- x inclusive DIS data. A detailed comparison with earlier geometric scaling violating models of the dipole scattering amplitude in terms of an anomalous dimension γ is made. In order to establish whether the geometric scaling violations expected from small- x evolution equations are present in the data a much larger range in transverse momentum and rapidity must be probed. Predictions for hadron production in p -Pb and p -p collisions at LHC are given. We point out that the fall-off of the transverse momentum distribution at LHC is a sensitive probe of the variation of γ in a region where x is much smaller than at RHIC. In this way, the expectation for the rise of γ from small- x evolution can be tested.

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I. INTRODUCTION

The observed phenomenon of geometric scaling, i.e. the property that the small- x DIS cross section depends only on x and Q^2 through the combination $Q^2/Q_s^2(x)$, where $Q_s(x)$ is referred to as the saturation scale, still requires a satisfactory explanation. It is a property that appears in a natural way in the (asymptotic) solutions of nonlinear evolution equations, such as the GLR equation [1, 2] or the BK equation [3, 4], that are expected to become relevant at small x . Nevertheless, the question remains whether the observed DIS data are obtained at sufficiently small x values for such evolution equations to be applicable. Here we want to investigate an extension of this question to the RHIC d -Au data and the future LHC p -Pb and p -p data. Like the DIS cross section, the hadron production cross sections in nucleon-nucleus scattering at high energies have been expressed in terms of the scattering of a color dipole off small- x partons, which are predominantly gluons [5, 6]. This dipole scattering amplitude is the quantity that is expected to display geometric scaling¹ and therefore should be a function of $Q^2/Q_s^2(x)$. In nucleon-nucleus collisions the role of Q is played by the transverse momentum q_t of the produced parton that fragments into the observed hadron (or by the inverse of its Fourier conjugate r_t). For earlier works about d -Au collisions and saturation physics we refer to Refs. [7, 8, 9, 10, 11] and the review [12].

A successful phenomenological study of experimental DIS data using a model for the dipole cross section was performed by Golec-Biernat and Wüsthoff (GBW) [13]. They found that the HERA data on the structure function F_2 at low x ($x \lesssim 0.01$) could be described well by a dipole cross section of the form $\sigma = \sigma_0 N_{GBW}(r_t, x)$, where $\sigma_0 \simeq 23$ mb and the scattering amplitude N_{GBW} is given by

$$N_{GBW}(r_t, x) = 1 - \exp\left(-\frac{1}{4}r_t^2 Q_s^2(x)\right). \quad (1)$$

This amplitude depends on x and r_t (the transverse size of the dipole) only through the combination $r_t^2 Q_s^2(x)$, which means it is geometrically scaling. The x -dependence of the saturation scale is given by

$$Q_s(x) = 1 \text{ GeV} \left(\frac{x_0}{x}\right)^{\lambda/2}, \quad (2)$$

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¹ More precisely, in momentum space Q^2 times the scattering amplitude is the scaling dimensionless quantity.

with $x_0 \simeq 3 \times 10^{-4}$ and $\lambda \simeq 0.3$. For nuclear targets Q_s^2 contains an additional factor $A^{1/3}$.

Model independent analyses of the HERA data show that the low- x data display geometric scaling for all Q^2 [14, 15], even though the GBW model, (1)-(2), was found to be inconsistent with newer, more accurate data at large Q^2 and requires modification. In Ref. [16] such a modification was proposed which includes DGLAP evolution, in order to fit the $Q^2 > 20 \text{ GeV}^2$ data. In Ref. [17] the impact of DGLAP evolution on a geometric scaling solution has been numerically studied. An initial condition was constructed so that at $Q^2 = Q_s^2(x)$, with $Q_s(x)$ as in Eq. (2), the dipole cross section at leading order (and hence $\alpha_s(Q^2)xg(x, Q^2)/Q^2$) is a constant as required for a geometric scaling solution. It was found that under DGLAP evolution to higher values of Q^2 geometric scaling is not violated for $\lambda \geq 4N_c\alpha_s/\pi$ in the fixed coupling constant case and only mildly violated for all values of λ in the running coupling constant case. In the latter case geometric scaling (GS) holds to very good approximation in the region $\log Q^2/Q_s^2 \ll \log Q_s^2/\Lambda^2$ (here Λ denotes Λ_{QCD}). One can conclude that although the DGLAP evolution equation for the gluon distribution does not necessarily lead to a GS solution itself, this GS property can be preserved to a large extent. The DGLAP induced violations of GS can remain small over a wide range of Q^2 values.

Similar studies have been performed for the BFKL equation [18, 19]. The solution of the BFKL equation with an appropriate boundary condition at Q_s was found to be geometrically scaling in leading order in the saddle point approximation [20, 21, 22, 23]. Beyond leading order for $1 \lesssim \log Q^2/Q_s^2 \ll \log Q_s^2/\Lambda^2$ this solution shows approximate scaling. At the scale $Q_{gs} \equiv Q_s^2/\Lambda$ the violations of geometric scaling are considered sizeable. This of course assumes that the BFKL equation governs the evolution in this entire region that has been called the extended geometric scaling (EGS) region. The region $Q^2 < Q_s^2$ is referred to as the saturation region.

The EGS region need not be equal to the region in which GS is observed in experiments, since it is unclear that the chosen evolution equation is appropriate in the entire region in the first place. But even if this is the case, one could not determine Q_{gs} from the data for a given rapidity. Only if one studies the data as function of Q^2/Q_s^2 for a range of rapidities (for which Q_{gs} is not a constant scale) will one be able to establish the extent to which GS is violated.

As mentioned, the DIS data for $Q^2 > 20 \text{ GeV}^2$ prompted the authors of Ref. [16] to propose a modification of the GBW model which includes DGLAP evolution. However, in Ref. [24] a model (IIM) has been put forward that is a modification of the GBW model and incorporates the violations of geometric scaling expected to arise from BFKL evolution in the EGS region. This model leads to a satisfactory fit to DIS data, but without the need to include DGLAP evolution at larger Q^2 . In Ref. [25] a description of the DIS data is obtained by taking into account both BK and DGLAP evolution. The fact that DIS data can be described using different approaches suggests that the small- x DIS data do not span a sufficiently large region in Q^2 and x to discriminate between the different types of evolution. The question is whether or not the RHIC and future LHC data do span a sufficiently large region.

In order to investigate GS violations in the RHIC data, in Refs. [26, 27] a phenomenological model, similar to the IIM model, has been put forward (following in part the earlier study of Ref. [28] based on [8]). We will refer to this model as the DHJ model. It offers a good description of the p_t distribution of hadrons produced in d - Au collisions at RHIC in the forward region ², and even in p - p collisions in the very forward rapidity region [29].

According to Refs. [26, 27] the cross section³ of single-inclusive forward hadron production in high-energy nucleon-nucleus collisions is described in terms of the dipole scattering amplitude in the following way,

$$\begin{aligned} \frac{dN_h}{dy_h d^2p_t} = & \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^1 dx_1 \frac{x_1}{x_F} \left[f_{q/p}(x_1, p_t^2) N_F \left(\frac{x_1}{x_F} p_t, x_2 \right) D_{h/q} \left(\frac{x_F}{x_1}, p_t^2 \right) \right. \\ & \left. + f_{g/p}(x_1, p_t^2) N_A \left(\frac{x_1}{x_F} p_t, x_2 \right) D_{h/g} \left(\frac{x_F}{x_1}, p_t^2 \right) \right]. \end{aligned} \quad (3)$$

A summation over quark flavors q is understood. Here N_F describes a quark scattering off the nucleus, while N_A applies to a gluon. The parton distribution functions $f_{q/p}$ and the fragmentation functions $D_{h/q}$ are considered

² As it turned out the central-rapidity study of Ref. [27] contained an error in the numerical code. The larger p_t data for $y_h = 0, 1$ are in fact not well-described by the DHJ model as will be seen.

³ To be more precise, Eq. (3) is an expression for the minimum bias invariant yield.

at the scale $Q^2 = p_t^2$, which we will always take to be larger than 1 GeV². The momentum fraction of the target partons equals $x_2 = x_1 \exp(-2y_h)$. We find that for pion, kaon, proton and even Λ production we can to good approximation neglect finite mass effects, i.e. we equate the pseudorapidity η and the rapidity y_h and use $x_F = \sqrt{p_t^2 + m^2}/\sqrt{s} \exp(\eta) \approx p_t/\sqrt{s} \exp(y_h)$. Finally, there is an overall K -factor that effectively accounts for NLO corrections. As it is expected that these corrections are more important at small y_h , the K -factor is allowed to be y_h dependent. A NLO pQCD analysis of the process $pp \rightarrow \pi^0 X$ at mid-rapidity for RHIC energies shows that such K -factors are relatively constant with p_t [30].

The dipole scattering amplitude of the DHJ model is given by [26, 27]:

$$\begin{aligned} N_A(q_t, x_2) &\equiv \int d^2 r_t e^{i\vec{q}_t \cdot \vec{r}_t} N_A(r_t, q_t, x_2) \\ &\equiv \int d^2 r_t e^{i\vec{q}_t \cdot \vec{r}_t} \left[1 - \exp \left(-\frac{1}{4} (r_t^2 Q_s^2(x_2))^{\gamma(q_t, x_2)} \right) \right]. \end{aligned} \quad (4)$$

Note that γ is a function of q_t rather than r_t . This allows one to compute the Fourier transform more easily. The corresponding expression N_F for quarks is obtained from N_A by the replacement $(r_t^2 Q_s^2)^\gamma \rightarrow ((C_F/C_A) r_t^2 Q_s^2)^\gamma$, with $C_F/C_A = 4/9$. The exponent γ is usually referred to as the “anomalous dimension”, although the connection between $N_{A/F}$ and the gluon distribution inside the nucleus cannot always be made.

The anomalous dimension of the DHJ model is parameterized as

$$\gamma(q_t, x_2) = \gamma_s + (1 - \gamma_s) \frac{\log(q_t^2/Q_s^2(x_2))}{\lambda y + d\sqrt{y} + \log(q_t^2/Q_s^2(x_2))}, \quad (5)$$

where $y = \log 1/x_2$ is minus the rapidity of the target parton. The saturation scale $Q_s(x_2)$ and the parameter λ are taken from the GBW model, as given in Eq. (2). Here Q_s includes the additional factor $A^{1/3}$, for which DHJ use $A_{\text{eff}} = 18.5$ in the d -Au case. The parameter d was fitted to the data and set to $d = 1.2$. This choice of γ leads to a geometric scaling solution at $q_t = Q_s$ where $\gamma = \gamma_s = 0.628$ and incorporates to a certain extent the violation expected from BFKL evolution for larger q_t . The anomalous dimension of DHJ is of the form $\gamma = \gamma_s + \Delta\gamma$, where the scaling violations arising from $\Delta\gamma$ behave as $\log(q_t^2/Q_s^2)/y$ for large y and $q_t^2 \gtrsim Q_s^2$ as resulting from the analyses of Refs. [21, 22, 23]. The question we will address in this paper is whether these violations are really seen in the available data. The fact that the DHJ model works well for forward hadron production in d -Au collisions does not demonstrate that there are actually violations present as we will show in detail.

For very large q_t , or equivalently for small r_t , one can use to good approximation

$$N_A(r_t, q_t, x_2) \approx \frac{1}{4} (r_t^2 Q_s^2(x_2))^{\gamma(q_t, x_2)}. \quad (6)$$

DHJ used the perturbative t -channel one-gluon exchange result to conclude that $\gamma \rightarrow 1$ as $q_t \rightarrow \infty$. However, as discussed in Ref. [31] also if the BFKL equation governs the large- q_t region, one can find that $\gamma \rightarrow 1$ at large q_t . The way in which γ approaches 1 directly determines how fast the cross section will fall off with increasing p_t as we will discuss in the next section.

In the DHJ model one retains GS approximately when $\Delta\gamma$ is small w.r.t. γ_s . For large, but fixed rapidity ($y \gg (d/\lambda)^2$) this holds numerically up to $q_t \approx Q_{gs} = Q_s^2/\Lambda$, when $\log Q_{gs}^2/Q_s^2 \sim \lambda y$. In general it is not simply the variation of γ that determines the GS violations. If γ is chosen to be a function of q_t^2/Q_s^2 or $r_t^2 Q_s^2$ only, the model is never GS violating no matter how fast it approaches 1 at large q_t . We will present such a scaling model below and demonstrate that it can describe the RHIC data in both the central and forward rapidity regions. Although the new parameterization of γ is similar in form to that of the DHJ model, it does not have the GS violating behavior nor the logarithmic rise expected from the BFKL (and more generally, BK) equation.

The outline of this paper is as follows. In Section II we discuss the properties of a new phenomenological model of the dipole cross section and the parameters fitted to RHIC data. A comparison to DIS data and the GBW model is also made. In Section III we present predictions for hadron and jet production at LHC. In Section IV we summarize our main conclusions.

II. NEW MODEL

We will now use RHIC data and the cross section expression in Eq. (3), as employed by DHJ [26, 27], to constrain the anomalous dimension γ entering the dipole cross section (4). Especially, we would like to address

the question whether the RHIC data really require violation of geometric scaling as it was claimed in Refs. [26, 27] and also stated in Ref. [32]. In DIS a scaling behavior of the dipole scattering amplitude $N(rQ_s)$ maps directly into a scaling of the DIS cross section $\sigma_{\gamma^* p}$, which is clearly observable in the data at small x ($x \lesssim 0.01$). Due to the convolution in Eq. (3) the situation is more involved in hadron-hadron collisions where no such scaling in terms of the observed kinematic variables (y_h and p_t) can be expected. Therefore we have to focus on the question whether hadron production in d - Au collisions is describable in terms of scaling dipole cross sections N_A and N_F .

An anomalous dimension γ leading to a geometric scaling dipole cross section should depend on $w = q_t/Q_s(x_2)$, but not separately on q_t and rapidity $y = \log 1/x_2$. We will take a value $\gamma_1 = \gamma(w = 1)$ of the order of γ_s and $\gamma \rightarrow 1$ for larger w . The parameterization that we adopted reads

$$\gamma(w) = \gamma_1 + (1 - \gamma_1) \frac{(w^a - 1)}{(w^a - 1) + b}. \quad (7)$$

The two free parameters a and b will be fitted to the RHIC data. The parameterizations (5) and (7) differ not only in the scaling behavior, but also the large q_t limit of $\gamma \rightarrow 1$ is approached much faster in the latter case. This will lead to different large momentum slopes of the dipole scattering amplitude (4) and therefore to different predictions for the large p_t slope using Eq. (3). For larger $w = q_t/Q_s$ the exponent can be expanded and the dipole scattering amplitude (4) simplifies, cf. Eq. (6) (we will suppress the possible y dependence),

$$N_A(q_t) \approx \frac{2\pi}{q_t^2} \frac{1}{w^{2\gamma(w)}} \frac{1}{4} \int_0^\infty dz z J_0(z) (-z^{2\gamma(w)}) = \frac{2\pi 2^{2\gamma(w)-1} Q_s^{2\gamma(w)}}{q_t^{2\gamma(w)+2}} \frac{\Gamma(1 + \gamma(w))}{-\Gamma(-\gamma(w))} \quad (8)$$

$$\stackrel{\gamma(w) \rightarrow 1}{\approx} \frac{4\pi Q_s^2}{q_t^4} (1 - \gamma(w)) \propto \begin{cases} \frac{Q_s^2}{q_t^4 \log(q_t^2/Q_s^2)} & \text{for } \gamma \text{ of Eq. (5)} \\ \frac{Q_s^{2+a}}{q_t^{4+a}} & \text{for } \gamma \text{ of Eq. (7)} \end{cases}.$$

For a constant $\gamma < 1$ the amplitude will drop even more slowly than both these models, namely $\propto Q_s^{2\gamma}/q_t^{2\gamma+2}$. For $\gamma = 1$ (the GBW model) one finds on the other hand an unrealistic exponential fall-off $\propto \exp(-q_t^2/Q_s^2)/Q_s^2$, which could be corrected by including a GS violating logarithm as in the MV model [33]. Due to the convolution in Eq. (3) with the parton distribution and fragmentation functions, the slope of the p_t distribution is not so simple to estimate. Empirically we find that the power of the p_t distribution is roughly a factor of one to two larger than the power of the dipole scattering amplitude. Below we are going to determine this power. We emphasize that the fall-off with p_t is not determined by the size of the scaling violations. In order to observe such violations one has to study both the y_h and p_t dependence over a significantly large range. Moreover, the scaling properties of the dipole scattering amplitude are not directly visible in the hadron production data, due to the parton distribution and fragmentation functions.

A. Comparison with RHIC data

In Fig. 1 we show our estimate for $dN_h/(dy_h d^2p_t)$ that follows from the integral in Eq. (3) with our parameterization for $\gamma(w)$ (7), which enters the dipole scattering amplitude (4). All p_t distributions of produced hadrons measured at RHIC in d - Au collisions [34, 35, 36] are well described. At the saturation scale we have chosen here for γ the same value $\gamma_s = 0.628$ as in the DHJ model. We also take $A_{\text{eff}} = 18.5$. We obtain the best fit of the data for:

$$a = 2.82 \quad \text{and} \quad b = 168. \quad (9)$$

As mentioned, this LO analysis requires the inclusion of a K -factor to account for NLO corrections, which are expected to become more relevant towards central rapidity. Following DHJ, the K -factor is allowed to vary with y_h , but is demanded to be p_t independent. The K factors we obtain for $y_h = 0, 1, 2.2, 3.2, 4$ are for our new model equal to $K = 3.4, 2.9, 2.0, 1.6, 0.7$ and for the DHJ model $K = 4.3, 3.3, 2.3, 1.7, 0.7$. We have assumed isospin invariance to obtain the parton distributions for a deuteron from those for a proton, using the CTEQ5-LO ones [37]. Furthermore, we use the KKP fragmentation functions of Ref. [38].

From this analysis we can conclude that a GS dipole scattering amplitude is completely compatible with the data and therefore the conclusion that GS violations are observed at RHIC cannot be drawn. Of course, a scaling

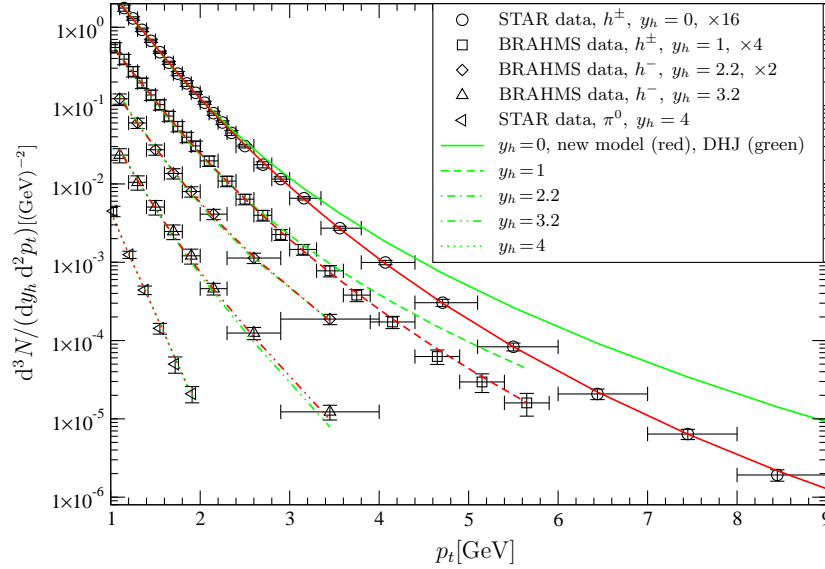


FIG. 1: Transverse momentum distribution of produced hadrons in d -Au collisions as measured at RHIC (black symbols) for various rapidities y_h . Using the scaling parameterization (7) the data are well described by the expression (3) with an appropriate K -factor (red/dark curves). The DHJ model (5) works well only for smaller p_t (green/light curves). To make the plot clearer, the data and the curves for $y_h = 0, 1$ and 2.2 are multiplied with arbitrary factors, namely $16, 4$ and 2 , respectively. The STAR data at $y_h = 0$ are from Ref. [34] and $y_h = 4$ from [36]. The BRAHMS results for $y_h = 1 - 3.2$ can be found in [35].

violating amplitude, i.e. a γ that depends on w and the rapidity y explicitly, is not ruled out by the data either. What can be concluded further is that the logarithmic rise of γ resulting from the BFKL evolution incorporated in the DHJ model is ruled out in the central region, see Fig. 1. This may simply indicate that x_2 is already so large that one is in the DGLAP region. In Fig. 2, the kinematic region where x_2 is small is indicated in terms of the observables p_t and y_h . Where the DHJ model starts to deviate from the data x_2 becomes larger than 0.01 , although Q_s is still larger than in DIS at $x = 0.01$. If one were to exclude the central rapidity RHIC data in the model fit, one could also obtain a scaling model with a logarithmically rising, or even constant, γ .

To indicate how much γ is constrained by the RHIC data, Fig. 3 shows various $\gamma(w)$'s that describe the available data equally well. They are all parameterized as in Eq. (7) with different a and b values, but require different K factors. Clearly, γ is less well determined close to the saturation scale than in the dilute region. This is because the integrand entering the dipole scattering amplitude (4) around the saturation scale $r = 1/Q_s$ is only weakly dependent on γ . In addition, the forward data ($y_h = 3.2$ and 4) are essentially sensitive only to γ_1 , since they probe the region where w is close to 1 . Therefore, the rise of γ with w is effectively constrained only by the data for $y_h = 0, 1$.

It is important to realize that given a non-scaling $\gamma(w, y)$ that fits the data for some value of y_h one can always find a scaling $\tilde{\gamma}(w)$ that leads to the same p_t distribution. This may not be obvious since even if y_h is fixed, a range of y values is probed in the convolution integral (3). However, the scaling parameter w can always be expressed as a function of y and y_h ,

$$w = \frac{x_1}{x_F} \frac{p_t}{Q_s} = x_2 \exp[y_h] \frac{\sqrt{s}}{Q_s} = \exp[-y + y_h] \frac{\sqrt{s}}{Q_s(y)} = w(y; y_h, s). \quad (10)$$

Hence, if y_h is kept fixed one can express the rapidity y in terms of w and define a scaling $\tilde{\gamma}(w) \equiv \gamma(w, y = y(w))$ that leads to the same results as $\gamma(w, y)$. Clearly, without probing a sufficiently large range of y_h scaling violations cannot be established.

As mentioned before, γ is chosen to be a function of q_t rather than r_t . For completeness it should be said that it is possible to describe the data equally well with a scaling γ that depends on $r_t Q_s$. In general, this will be a different function than one would obtain by simply replacing q_t with $1/r_t$ in γ which would lead to unphysical oscillations in the dipole scattering amplitude and hence in the hadron production cross section.

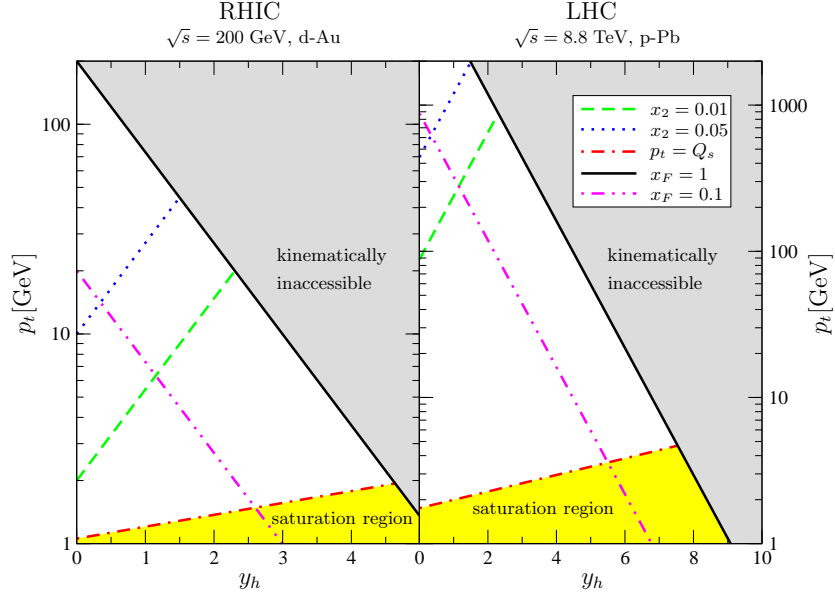


FIG. 2: Illustration of the kinematical ranges relevant for RHIC and LHC. The saturation region is set by the line $q_t = x_1/x_F p_t = Q_s(x_2)$. Since the dominant contribution to (3) comes from the region x_1 close to $x_F = p_t/\sqrt{s} \exp(y_h)$, we used for this plot $x_2 = x_1 \exp(-2y_h) \approx p_t/\sqrt{s} \exp(-y_h)$ and $x_1 \approx x_F$. For d -Au we have taken $A_{\text{eff}} = 18.5$ and for p -Pb 20. The curves of constant x_2 indicate the regions where small- x physics may become relevant.

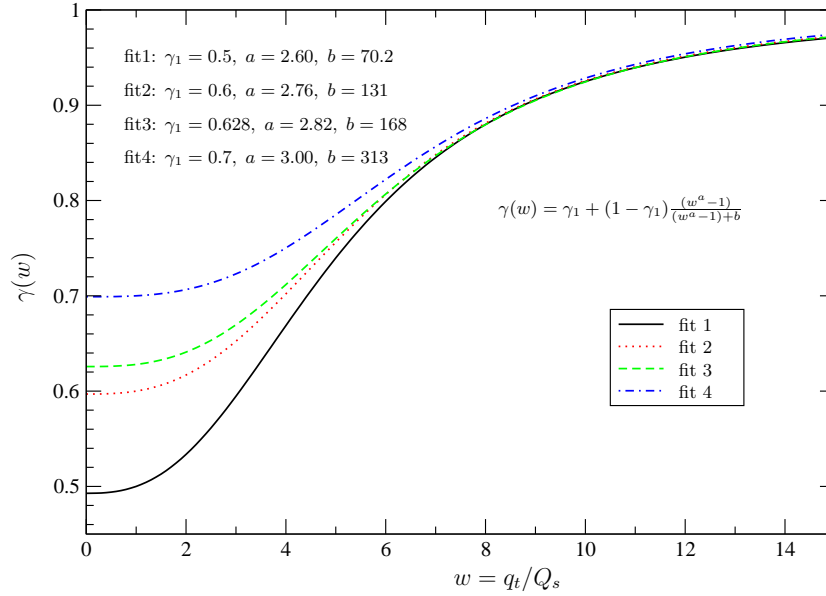


FIG. 3: Various fits of $\gamma(w)$, which describe the RHIC data equally well. For choices of γ outside this range the data are less well described.

B. Compatibility with deep-inelastic scattering

Since the parameterization (4) of the dipole scattering amplitude uses an anomalous dimension $\gamma \neq 1$, the resulting amplitude is quite different from the GBW model. Therefore, it is important to check whether our anomalous dimension $\gamma(w)$ is still compatible with the DIS data. For this we use the following expression for the

dipole scattering amplitude

$$N_\gamma(r_t, Q, x) = 1 - \exp\left(-\frac{1}{4}(r_t^2 Q_s^2(x))^{\gamma(w=\sqrt{Q^2/Q_s^2(x)})}\right), \quad (11)$$

where Q_s is given by Eq. (2) and for γ we use our model, Eqs. (7) and (9).

Following the procedure in [13], we predict the total cross section $\sigma_{\gamma^*p} = \sigma_T + \sigma_L$ by folding the resulting dipole cross section $\sigma = \sigma_0 N_\gamma$ with the perturbatively calculable photon wave function,

$$\sigma_{T,L}(x, Q^2) = \int dz \int d^2 r_t |\psi_{T,L}(z, r_t, Q^2)|^2 \sigma(r_t, x), \quad (12)$$

where z is the longitudinal momentum fraction of the quark in the dipole.

In Fig. 4 we show the small- x HERA data [39, 40, 41] in a large kinematic range as a function of $\tau = Q^2/Q_s^2(x)$. Following Ref. [16], we scale the H1 data [39] by a factor 1.05, which is consistent with the normalization uncertainty. As can be seen, the data for $x < 0.01$ depend on x and Q^2 only through the variable τ . In Fig. 4 we compare these data with the original GBW model and the prediction following from our modified γ obtained from a fit to RHIC data. For both models we have neglected effects from finite quark masses in the photon wave function, which break geometric scaling. As a result the cross section of the GBW model overshoots the data at small τ , i.e. at small Q^2 . Using the modified γ this effect and therefore the fitted quark mass is smaller since the smaller value of γ in the saturation region suppresses the cross section. Further details of the small- τ behavior can be found in e.g. [42]. In addition, we use a somewhat smaller σ_0 value in order to obtain a better description of the data. No parameters of γ are tuned. The smaller value of σ_0 is forced by the region $\tau \approx 10 \dots 100$ where γ is not yet close to one but the effective value of $r_t Q_s$ is already large. Given the normalization uncertainty of our model, we do not consider the smaller value of σ_0 a problem. Of course, it would be possible to obtain an optimized parameterization of γ by a simultaneous fit to the RHIC and DIS data. But at this stage we conclude that the model, which we constructed to describe the RHIC data, can describe the DIS data equally well as the GBW model if one adjusts the additional parameter σ_0 .

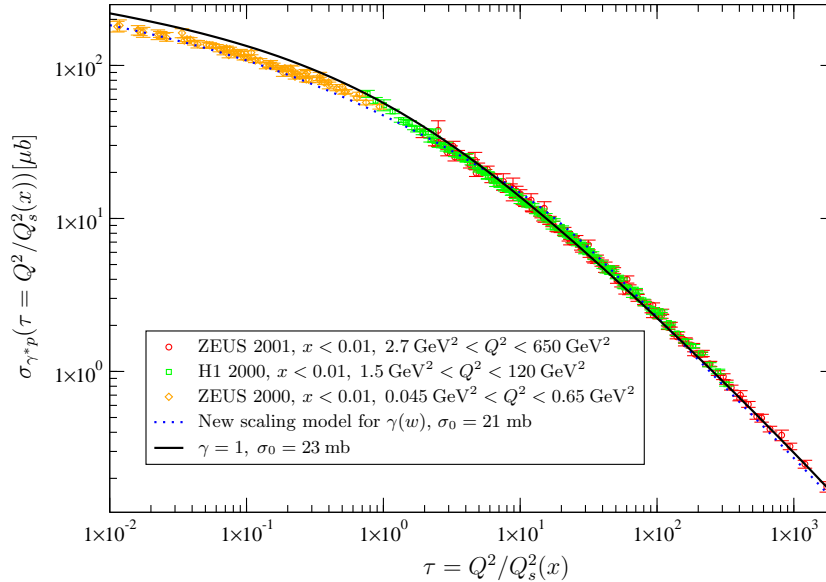


FIG. 4: The γ^*p cross section as a function of the scaling variable τ measured at HERA [39, 40, 41]. We compare the data with the predictions following from the original GBW model, where $\gamma = 1$, and the modified version fitted to RHIC data, where γ depends on $Q^2/Q_s^2(x)$.

We end this section with a comment on whether or not the factor (C_F/C_A) should have been included in N_γ of Eq. (11), as was done for N_F earlier. In order to compare the DHJ model or our new model with the GBW model, it would indeed be better to use Eq. (4) as a model for N_F and scale $(r_t^2 Q_s^2)^\gamma \rightarrow ((C_A/C_F)r_t^2 Q_s^2)^\gamma$ to

obtain N_A . A fit to RHIC data would then result in a somewhat different γ . Since this is not done by DHJ and we are specifically interested in a comparison to the DHJ model, we follow DHJ's approach. This does however obscure the comparison to the GBW model somewhat, since that is a model for N_F without the factor C_F/C_A . Note that for models with $\gamma \neq 1$ this cannot be accounted for by rescaling Q_0 , because one does not scale Q_s that enters in γ . In a future combined fit to RHIC and DIS data one would of course like to avoid this slight conceptual discrepancy.

III. LHC PREDICTIONS

A. Hadron production

We have seen that where the DHJ model curves deviate from the RHIC data, the x_2 -values probed are not very small. However, at LHC due to the much higher energies, the region of small x_2 extends to a much larger range of p_t , so that the predictions of the DHJ model and the new model will be different even at small x_2 . In Fig. 2 the region of small x_2 is depicted in terms of p_t and y_h for p - Pb collisions at LHC. In this section we discuss the predictions following from the DHJ model and our new scaling model for the hadron production cross section for p - p collisions at $\sqrt{s} = 14$ TeV and for p - Pb collisions at $\sqrt{s} = 8.8$ TeV.

In Fig. 5 we show the predictions for the p - p collisions at $\sqrt{s} = 14$ TeV. For smaller p_t the predictions of the DHJ model and the new model are comparable. This can be expected since this region corresponds to the small- p_t region at RHIC. For larger rapidities, i.e. $y_h \approx 7 - 8$, the predictions are indistinguishable since the reachable momenta $q_t \leq \sqrt{s} \exp(-y_h)$ are so small that the ratios $w = q_t/Q_s$ are always so close to one that γ is effectively equal to γ_s . However, there is quite a large range where the probed values of $x_2 \sim p_t/\sqrt{s} \exp(-y_h)$ are small but the predictions are clearly different. The slope of the cross section is much larger when described in our model as compared with the DHJ model, since γ rises towards 1 much faster. Hence, a measurement of the slopes at moderate rapidities y_h at LHC would allow a discrimination between the DHJ model and our model in a region where small- x physics may be expected to be applicable. Since a logarithmic rise of γ is a generic signature of BFKL evolution, these measurements offer the possibility of testing whether such small- x evolution is actually relevant at present-day hadron colliders.

The p - Pb predictions for LHC are very similar. However, due to the smaller energy of $\sqrt{s} = 8.8$ TeV the predictions are already comparable for smaller rapidities, i.e. for $y_h \approx 6$, cf. Fig. 6. Here the rapidities are given for the nucleon-nucleon center of mass frame, which for LHC is not the lab frame in contrast to RHIC. This means that in terms of rapidities in the lab frame there is a slight offset of $\Delta y_h = y_{\text{lab}} - y_{\text{cm}} \approx 0.47$ to take into account.

Note that for the whole kinematic range depicted in Figs. 5 and 6 the x_2 values are well below 0.01.

B. Jet production

Unlike in the case of the DIS cross section, geometric scaling of the dipole amplitude does not lead to scaling of the hadron production cross section at RHIC or LHC, because of the convolution of the amplitude with the non-scaling parton distributions and fragmentation functions. This effect can be reduced by considering jet production. The description of the jet cross section does not involve any fragmentation functions, but reduces to just a sum over products of dipole amplitudes and parton distribution functions,

$$\frac{dN_h}{dy_h d^2p_t} = \frac{K(y_h)}{(2\pi)^2} \left[\sum_q f_{q/p}(x_F, p_t^2) N_F(p_t, x_2) + f_{g/p}(x_F, p_t^2) N_A(p_t, x_2) \right], \quad (13)$$

where $x_F = p_t/\sqrt{s} \exp(y_h)$ and $x_2 = x_F \exp(-2y_h) = p_t/\sqrt{s} \exp(-y_h)$. This means that in the kinematical regions where either the gluon contribution or the quark contribution is dominant (in general in a small kinematical region), the corresponding distribution function can be divided out, so that one obtains the dipole amplitude directly from the data. Of course, if the dipole amplitude is only mildly scaling violating, this kinematical region may be too small to observe the violations in this way. At LHC, the gluon contribution to the jet cross section is reasonably dominant for transverse momenta $p_t \lesssim 15$ GeV and hadron rapidities $y_h = 0 - 2$. In this region, the scaling violations of the ratio $(p_t^2 dN_h/dy_h d^2p_t)/f_{g/p}(x_F, p_t^2)$ are in the DHJ model about 30%, while the violations for the exactly scaling model (7) are, due to quark contributions, still about 10%. We conclude that

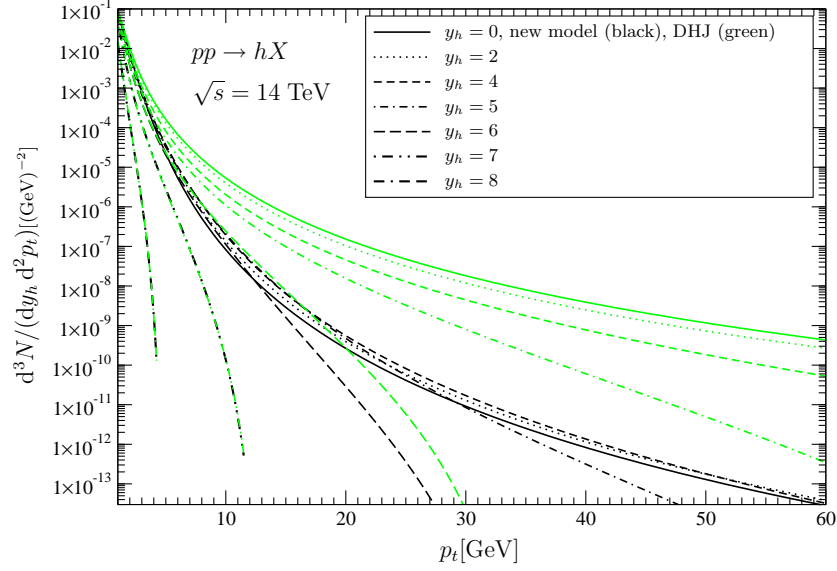


FIG. 5: Predictions of the transverse momentum distributions of produced hadrons in p - p collisions at the LHC energy of $\sqrt{s} = 14$ TeV and various rapidities $y_h = 0 - 8$. The distributions from the scaling model are represented by the black lines and those from the DHJ model by the green/light ones.

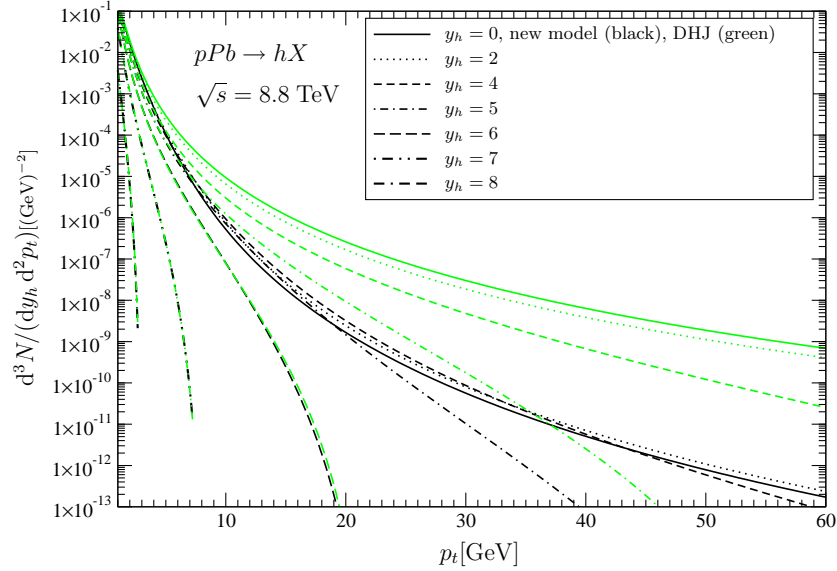


FIG. 6: Same as Fig. 5 but for p - Pb collisions at the LHC energy of $\sqrt{s} = 8.8$ TeV. We have used $A_{\text{eff}} = 20$.

it may be difficult to attribute any observed violations directly to N_A . A similar conclusion holds for N_F in the region where quarks dominate (when $x_F \gtrsim 0.1$, see Fig. 2).

In summary, even for jet production, where there are no complications from the fragmentation functions, it may not be possible to establish geometric scaling violations conclusively due to the mixture of quark and gluon contributions. The kinematic range at LHC where either quark or gluons dominate is probably too small to reach a definite conclusion about scaling violations.

IV. CONCLUSIONS

We have presented a new phenomenological model of the dipole scattering amplitude to demonstrate that the RHIC data for hadron production in d - Au collisions for all available rapidities are compatible with geometric scaling. Moreover, the model also provides a reasonable description of the small- x DIS data. On the other hand, in a region of y_h and p_t for which the probed values of x are sufficiently small, the RHIC data are also compatible with geometric scaling violating models, such as the DHJ model. The fact that the DHJ model, which incorporates scaling violations from BFKL (or more generally BK) evolution to some extent, also describes the forward RHIC data suggests that the data simply do not span a sufficiently large region in p_t and y_h to demonstrate possible violations of geometric scaling. Hence, it cannot be concluded that scaling violations of the dipole scattering amplitude play a role at RHIC.

The breakdown of the DHJ model at midrapidity might simply be due to the probed values of x being not sufficiently small. The situation is different at LHC in p - p and p - Pb collisions. For smaller rapidities, but still within the region where the small- x description could be applicable, the DHJ model and the new scaling model lead to different predictions for the p_t fall-off of the cross section. This fall-off is determined by how fast the anomalous dimension γ approaches 1 for large transverse momentum. BFKL evolution typically leads to a logarithmic rise of $\gamma \rightarrow 1$ with transverse momentum and therefore implies a fall-off that is much slower than one finds for the new scaling model that is compatible with both the RHIC and the DIS data. Therefore, at LHC in both p - p and p - Pb collisions the transverse momentum distribution will probe for the first time at sufficiently small x the rise of the anomalous dimension γ , and will thereby provide an important test of the expectations from small- x evolution.

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